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EXACT SOLUTION OF THE PROBLEM OF SUPERSONIC GAS FLOW PAST CERTAIN SPATIAL BODIES

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by

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EXACT SOLUTION OF THE PROBLEM OF SUPERSONIC GAS FLOW PAST CERTAIN SPATIAL BODIES *

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by A. L. Gonor

It was shown in [1, 2], that a star-like shape of the cross-section of any aircraft's frame allows to decrease by tens of times the wave drag by comparison with a spinning body of equivalent volume or midship section area. Because of the fact that the conclusions are based upon the utilization of the Newton formula for pressure distribution, it is necessary to ascertain whether or not such drag decrease is fictitious at the expense of error increase in the latter. The answer might be provided by comparison with the exact solution. However, only one exact solution is known for spatial bodies [3], whose application is restricted to pyramidal bodies, remote in their shape from the optimum ones. In the light of the above-said, it appears to be inresting to synthesize the exact solution for spatial bodies of star-like shape, similar to those studied in [1].

Let us consider a system of plane intersecting discontinuities passing through the origin of coordinates and defined by the angles α and γ (Fig. 1). We shall denote the velocity of the incident flow and the Mach number respectively by U and M_∞. We shall introduce the auxiliary angle γ_1 according to the formula $\operatorname{tg}\gamma_1 = \operatorname{tg}\gamma \sin\alpha$, ; then the velocity components behind the first discontinuity, referred to the

* Tochnoye resheniye zadachi obtekaniya nekotorykh prostranstvennykh tel sverkhzvukovym potokom gaza.

value of the velocity of the incident flow, are determined from the expressions

$$v_x = 1-(1-\varepsilon)(\sin^2\gamma_1 - M_\infty^{-2}),\quad v_y = (1-\varepsilon)\operatorname{ctg}\gamma_1(\sin^2\gamma_1 - M_\infty^{-2})\sin\alpha$$
$$v_z = -(1-\varepsilon)\operatorname{ctg}\gamma_1(\sin^2\gamma_1 - M_\infty^{-2})\cos\alpha,\quad \varepsilon = (\varkappa-1)(\varkappa+1)^{-1} \qquad (1)$$

($\varkappa$ being the adiabat index)

Let the total velocity vector, referred to the velocity U and the Mach number behind the first discontinuity be U_1 and M_1. The flow deflection upon passing through the first discontinuity (angle δ) may be computed from the correlation

$$\operatorname{tg}\delta = \operatorname{ctg}\gamma_1 \frac{(1-\varepsilon)(\sin^2\gamma_1 - M_\infty^{-2})}{1-(1-\varepsilon)(\sin^2\gamma_1 - M_\infty^{-2})}$$

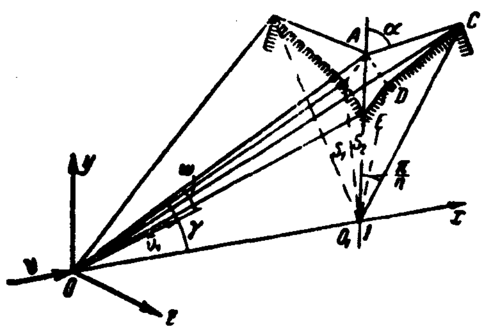

Fig. 1

After that, the Mach number M_1 is found by well known formulas for an oblique discontinuity. The perturbed gas flow behind the selected system of discontinuities will correspond to flow past a certain body, provided there is a regular intersection of discontinuities at the point A, permitting to rotate the flow parallelwise to symmetry plane. The necessary condition for a regular crossing is

$$M_{1n} = M_1 \sin\omega > 1 \qquad (2)$$

The angle ω between the velocity vector U_1 and the discontinuities intersection line OA is found from the correlation

$$\cos\omega\cos\gamma_1 = \cos(\gamma_1-\delta)\cos\gamma$$

Let us now introduce into the consideration the deflection angle θ of the flow, lying in a plane normal to the rib OA and formed by projections of the segment O_1A and of the velocity vector U_1 on this plane. After rather simple transformations, we shall obtain for its value the formula

$$\cos\theta = (\operatorname{tg}\gamma - \operatorname{tg}\delta\sin\alpha)\,[\operatorname{tg}^2\gamma(1+\cos^2\alpha\operatorname{tg}^2\delta) + \operatorname{tg}^2\delta - 2\sin\alpha\operatorname{tg}\gamma\operatorname{tg}\delta]^{-1/2} \qquad (3)$$

The disposition of the second wave (OAD in Fig. 1) can now be determined by the angle β, lying in the plane of the angle θ, from the condition that the turn of the flow in this plane is given by the expression (3). As a result, the angle β is found by formulas of the oblique discontinuity in the form

$$\operatorname{tg}\theta = 2\operatorname{ctg}\beta\,\frac{M_{1n}^2\sin^2\beta - 1}{M_{1n}^2(\varkappa + \cos 2\beta) + 2} \tag{4}$$

If the problem's initial parameters M_∞, γ and α are such that the expression (4) has a solution relative to the angle β, the constructed gas flow will correspond to a flow past a certain spatial body with a transverse cross section constituted of straight line segments. Let us define its geometry (points E, D, C, in Fig. 1). After elementary transformations, we find that points E and D have for coordinates

$$y_E = (\sin\alpha - \lambda\cos\alpha)\operatorname{tg}\delta, \qquad \lambda\,[\sin\gamma\cos\alpha\sin\delta - \operatorname{ctg}(\beta-\theta)\cos\delta] = \cos\omega$$
$$y_c = z_c\operatorname{ctg}(\pi/n), \qquad z_c\sin(\alpha - \pi/n) = \operatorname{tg}\gamma_1\sin(\pi/n) \tag{5}$$

For the determination of the position of the remaining points we must know the coordinate of the point F, lying over the extension of the wall CD. The calculation leads to the expression

$$y_F = \frac{\operatorname{tg}\gamma_1\operatorname{tg}\delta\cos(\alpha - \pi/n)}{\sin(\pi/n)\operatorname{tg}\gamma_1 + \sin(\alpha-\pi/n)\cos\alpha\operatorname{tg}\delta} \tag{6}$$

Hence, we find that the coordinates of the point D can be found from the formulas

$$y_D = \frac{y_F\operatorname{ctg}(\beta-\theta) + \sin\gamma\operatorname{ctg}(\alpha-\psi)}{\operatorname{ctg}(\beta-\theta) + \cos\gamma\operatorname{ctg}(\alpha-\psi)}$$
$$z_D = \frac{\sin\gamma - \cos\gamma\, y_F}{\operatorname{ctg}(\beta-\theta) + \cos\gamma\operatorname{ctg}(\alpha-\psi)} \tag{7}$$
$$\operatorname{tg}\psi = \left(1 - \frac{\operatorname{tg}\delta}{\operatorname{tg}\gamma_1}\right)\operatorname{tg}\left(\alpha - \frac{\pi}{n}\right)$$

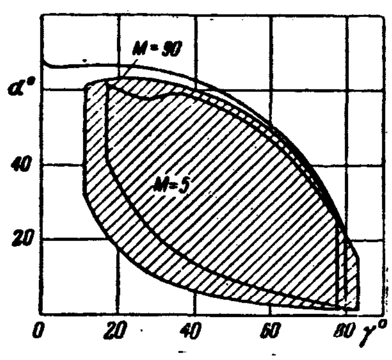

Fig. 2

The projections of the regions of perturbed flow behind the first and second discontinuities on the plane $x = 1$ are determined by the areas S_1 and S_2. Their values, using (5) – (7), are computed from the correlations

$$2S_1 = y_F(z_C - z_D), \qquad 2S_2 = y_E z_D$$

Let us now pass to the computation of forces. The wall CD is subject to a force caused by the constant pressure

$$c_p^{(1)} = 2\,(1-\varepsilon)\,(\sin^2\gamma_1 - M_\infty^{-2})$$

This wall ED is situated in a region of increased pressure, the value of which being

$$c_p^{(2)} = c_p^{(1)} + 2U_1^2\,\frac{\sin^2\omega\,(\sin^2\beta - M_{1n}^{-2})\,(1-\varepsilon)\,M_\infty^2\sin^2\gamma_1}{1+\varepsilon\,(M_\infty^2\sin^2\gamma_1 - 1)}$$

In each of the regions ACD and ADE (Fig. 1) the flow is uniform. All the lines of current in the plane $x = 1$, passing through the discontinuity AC, converge into one point E, which is the Ferri point for this flow about. At the same time, during the passage of the second wave AD, all the current lines, including the wall have a break. The wave drag of the considered body is represented in the form

$$C_x = \frac{c_p^{(1)}S_1 + c_p^{(2)}S_2}{S_1 + S_2}$$

The solution of the inverse problem exists only for a specific range of values of the parameters M_∞, α, γ and n. For example, the inequality $\alpha > \pi/n$ must be always fulfilled. Obviously, other limitations exist also. One of such limitations is the necessary condition (2). Its utilization provides for every value of the Mach number of the incident flow a specific region of acceptable values of the parameters α and γ. The results of computation for the range of numbers $M_\infty = 5$, 10 and ∞ at adiabat index $\varkappa = 1.4$ are shown in Fig. 2. The corresponding regions are shaded. It must be borne in mind that the condition (2) is not sufficient, and that is why the choice of parameters within the indicated regions has a character of preliminary selection. The computation of the dependence of wave drag on the angle α for certain values of the parameters M_∞, n and γ ($\varkappa = 1.4$) is shown in Figs. 3 – 4.

The value C_x^0/C_x, plotted in the ordinate axis, represents the drag ratio of a circular cone, equivalent by length and midship section area to that of a star-shaped body.

The computation for $M_\infty = \infty, \gamma = 10^\circ$ and 20° is executed in Fig. 3. As may be seen from the graph, the difference in the drag decreases as the angle α increases. However, the drag of a star-shaped body still remains by about 10 times smaller even at great values of α, than for an equivalent cone. As the value $\alpha (\alpha \to \pi/n)$ decreases, the drag of a star-shaped body drops, and difference with the cone increases. Comparison of the curves of Fig. 3 shows that the drag of the body will

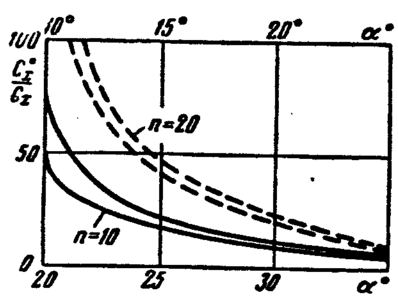

FIG. 3.

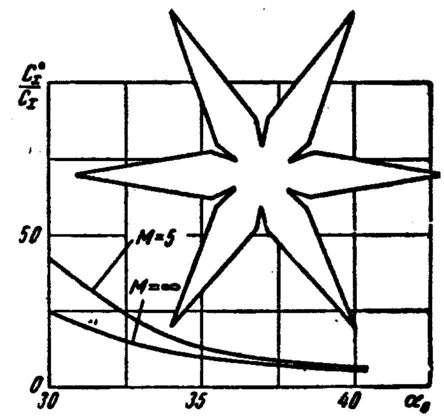

FIG. 4.

decrease with the increase of the number of ribs n with all other parameters unchanged, whereas with the increase of the parameter γ it increases. An analogous qualitative behavior is preserved also for finite numbers of M_∞.

Let us point to a peculiarity in the disposition of the curves in Fig. 4. According to the graphs, the drag coefficient increases with the increase of M_∞, the other parameters remaining invariable. In reality various bodies are obtained at different Mach numbers, while the variation in body geometry is more manifest on the drag, than the change of the Mach number. The form of cross section for a single case at parameter values $n = 6, \gamma = 5^\circ, \alpha = 41^\circ$ and $M_\infty = \infty$ is also shown in Fig. 4. Alongside with the precise calculation of the wave drag, a computation by the Newton formula was made for the obtained bodies in a series of cases. The comparison disclosed an error of the order of 20 percent. Therefore, all the results on the significant decrease of wave drag, obtained earlier for star-shaped bodies [1 – 2]. agree well with the exact solution in both, qualitative and quantitative reference.

Note in conclusion, that the assumption on the possibility of similar solution was expressed independently by G. I. Maykapar [4].

*** THE END ***

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REFERENCES

[1].- A. L. GONOR.- PMM, T.27, vyp. 1, 1963.

[2].- G. G. CHERNYI, A. L. GONOR.- The determination of body shapes of minimum drag using Newton and Busemann pressure laws.- Paper presented at Symposium on Extremal Problems in Aerodynamics.- BSRL, Seattle, Wash., 1962.

[3].- G. I. MAYKAPAR.- PMM, T. 23, vyp. 2, 1959.

[4].- G. G. CHERNYY.- PMM, T. 28, vyp. 2, 1964.

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